Teaching materials
Guide notes 1. Platform identification

MISCE project

Mechatronics for Improving and Standardizing Competences in Engineering

Competence: Control Engineering

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Universitat Politècnica de València

This document corresponds to the first lecture for the competence ‘Control Engineering’ using the ‘DC-motor control platform’

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1 Objective

The main objective of this lesson is to obtain the mathematical model of the experimental platform in terms of its transfer function.

Obtaining the mathematical model of a system is crucial to design proper control strategies which can be supported by simulations prior to an experimental implementation.

2 Mathematical model of a DC-motor

In order to describe the static and dynamic behaviour of a DC motor, it is necessary to establish a mathematical model. This model should be as simple as possible while accurately fitting reality. The model obtained below is a linear model in the Laplace domain. The first step in modelling the motor is to consider all variables that affect its behaviour. Figure 1 represents a schematized DC motor where the constants of the model are:

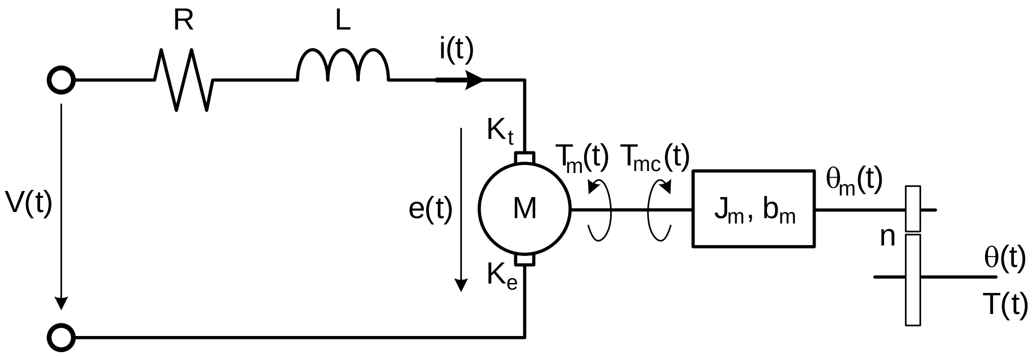


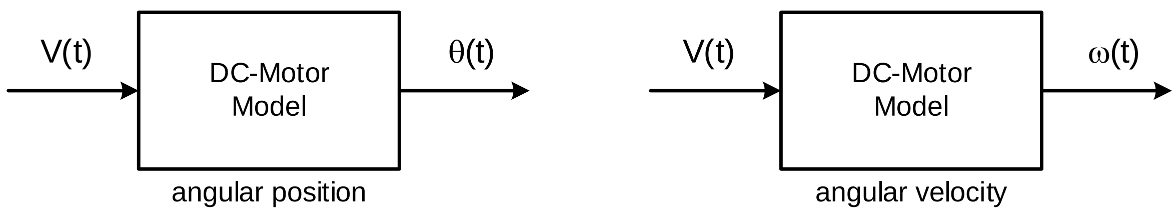
Figure 1. Scheme for the mathematical model of a DC motor.

- R is the electric resistance.
- L is the electric inductance.
- K_t is the motor torque constant.
- K_e is the electromotive force constant.
- J_m is the motor inertia.
- b_m is the viscous friction coefficient of the motor.
- n is the gearbox reduction ratio.

and the variables:

- $V(t)$ is the voltage input signal.
- $i(t)$ is the current.
- $e(t)$ is the motor voltage.
- $T_m(t)$ is the motor torque.
- $T_{mc}(t)$ is the friction torque at the motor.
- $\theta_m(t)$ is the angular position of the motor.
- $T(t)$ is the output shaft torque.
- $\theta(t)$ is the angular position of the output shaft.

In terms of the mathematical model for control, the input of the system is the voltage $V(t)$ and the output the angular position of the output shaft, $\theta(t)$, if used for position control, or its corresponding angular velocity, $\omega(t)$, if used for velocity control (see Figure 2).



a) b)

Figure 2. Block diagram of the DC-motor model. a) angular position; b) angular velocity.

The dynamic behaviour of the electrical part of the system can be described by:

$$V(t) = R \cdot i(t) + L\frac{di(t)}{dt} + e(t) \quad (1)$$

being the voltage of the motor:

$$e(t) = K_E \cdot \frac{d\theta_m(t)}{dt} \quad (2)$$

and being the torque exerted by the motor is:

$$T(t) = K_T \cdot i(t) \quad (3)$$

The dynamic of the mechanical part is given by:

$$T(t) = K_m \cdot v(t) = J_m \cdot \frac{d^2\theta_m(t)}{dt^2} + b_m \cdot \frac{d\theta_m(t)}{dt} + T_{mc}(t) \quad (4)$$

where K_m is an electromechanical constant.

The output variable is the angular position of the output shaft of the gear box which is related to the angular position of the motor as:

$$\theta(t) = \frac{\theta_m(t)}{n} \quad (5)$$

but the exerted torque of the output shaft is therefore:

$$T(t) = T_m(t) \cdot n \quad (6)$$

Rearranging these equations, the dynamics of the mechanical part can be written as:

$$K_m \cdot V(t) = J \cdot \frac{d^2\theta(t)}{dt^2} + b \cdot \frac{d\theta}{dt} + T_c(t) \quad (7)$$

being J and b the inertia and viscous friction at the gearbox output shaft and T_c its friction torque.

Term $T_c(t)$ is non-linear, so, if a linear model shall be obtained, it can be neglected. Applying Laplace transform, dynamics of the DC-motor can be described as:

$$K_m \cdot V(s) = J \cdot s^2 \cdot \Theta(s) + b \cdot s \cdot \Theta(s) \quad (8)$$

which yields to the following transfer function of the system:

$$G(s) = \frac{\Theta(s)}{V(s)} = \frac{A}{s(s+B)} \quad (9)$$

being $A = \frac{K_m}{J}$ and $B = \frac{b}{J}$.

Note that the dynamics of the electrical part has been neglected owing to its fast response in comparison to the mechanical part.

The identification procedure in the next section describes the steps to obtain the dynamics parameters A and B of the experimental platform detailed in the Lesson 0. Introduction to the experimental platform.

3 Identification procedure

3.1 Introduction

To perform the identification of the experimental platform, it must be excited by different known amplitude step inputs, $V(t)$, and record the system's response, $\theta(t)$. Since the experimental platform is equipped with an encoder, only the angular position can be recorded over time. To obtain the angular velocity over time, $\omega(t)$, it is necessary to numerically derive from the recorded angular position measurements.

Transfer function (9) can be rewritten for identification purpose as the relationship between the input voltage and the output angular velocity as:

$$G_{\omega}(s) = G(s) \cdot s = \frac{\Omega(s)}{V(s)} = \frac{A}{s+B} \quad (10)$$

Transfer function (10) is now a first order system that can be written in a normalised form as:

$$G_{\omega}(s) = \frac{K}{Ts+1} \quad (11)$$

being $K = \frac{A}{B}$ the static gain of the system and $T = \frac{1}{B}$ the time constant.

The dynamic of a first order system is well-known, and its step response is shown in Figure 3.

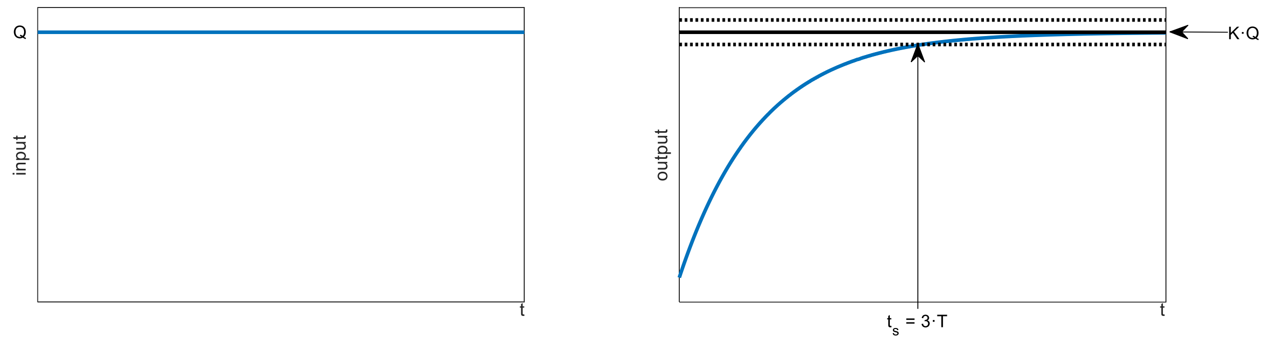


Figure 3. Step response of a first order system.

Note that, if the system is excited with a step input of amplitude Q, the final value of the response of the system is $K \cdot Q$. In this sense, the relationship between the amplitude of the input step and the final value of the response is K. On the other hand, the settling time[1] in the $\pm 5\%$ band is $3 \cdot T$.

Therefore, the procedure to identify the transfer function of the DC motor platform consists of exciting the system with different voltage step inputs, record the responses of the system and obtaining K and T.

[1] Settling time: The time required by the response to reach and steady within specified range of 5% of its final value.

3.2 Gain, K, identification

By exciting the system with different input voltage values, V_i, in the allowed range [-12, 12] V, the final value of the velocity of the system can be obtained, ω_i^{ss}.

For example, let's imagine a step input of amplitude 2 (Figure 4a) the DC-motor response shown in Figure 4b. For an input value of $V = 4\,V$, the steady state value of the angular velocity is $4.62\ rad/s$.

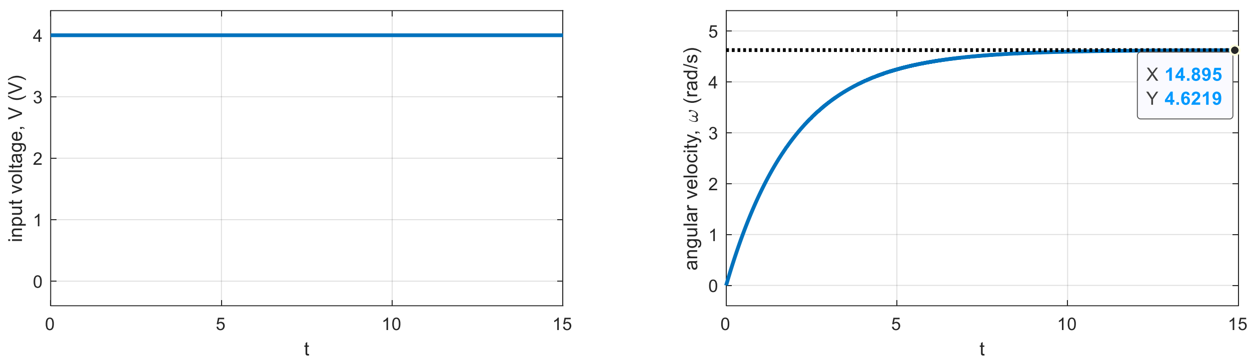


Figure 4. Example of experiment for plant identification.

By repeating this experiment for different values of input voltage, the relationship between these voltage inputs and the steady state angular velocity should be as in Figure 5. Note that a dead zone[2] and a saturation region[3] is expected.

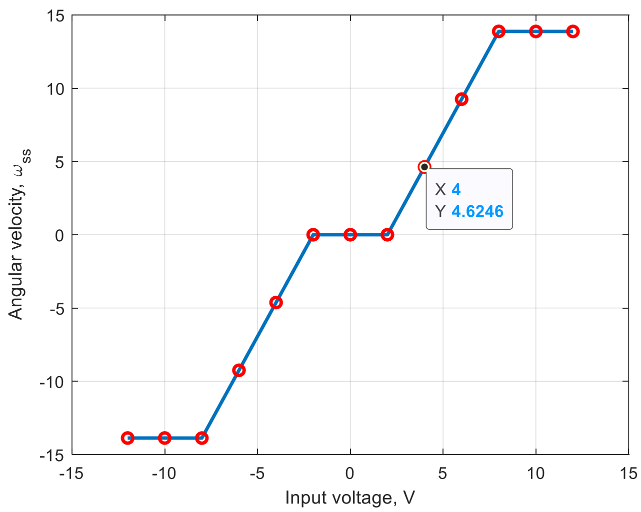


Figure 5. Relationship between input voltage and angular velocity

The slope, P, of the relationship between input voltage and angular velocity is:

$$P = \frac{\omega(\infty)}{V(\infty)} = \lim_{s \to 0} \frac{K}{Ts + 1} = K = \frac{A}{B} \quad (12)$$

[2] Dead zone: region where the input voltaje is small and motor does not move.
[3] Saturation region: region where the angular velocity reach its máximum/mínimum value that cannot be overpassed.

which has been illustrated in Figure 6.

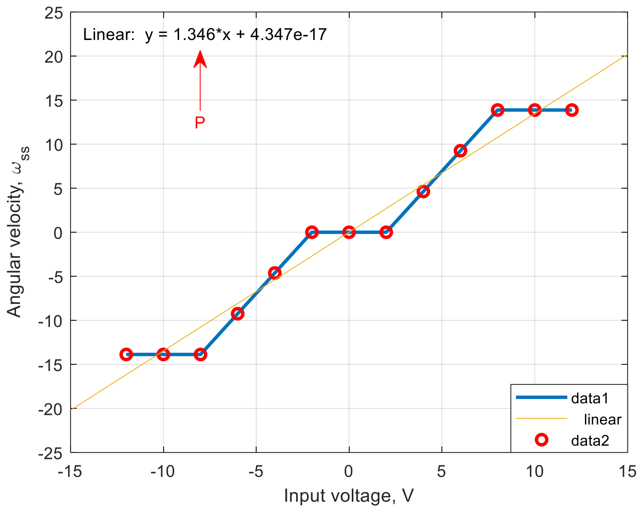


Figure 6. Identification of static gain, K.

3.3 Time constant, T, identification

To identify the time constant the equation of the settling time for a first order system is used:

$$t_s = 3 \cdot T \rightarrow T = \frac{t_s}{3} \quad (13)$$

In this sense, any of the dynamic time response of the previous experiments can be used to determine the settling time and, consequently, the time constant.

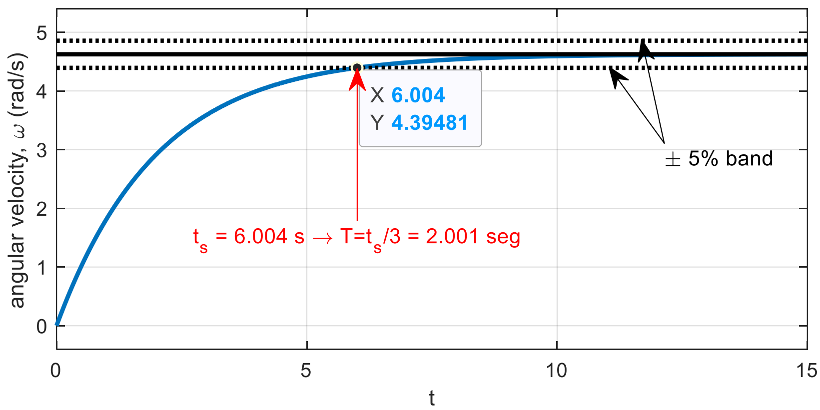


Figure 7. Identification of time constant, T.

With the determination of static gain, K, and time constant, T, the transfer function which model the dynamic behaviour of the system is completely identified.

$$G(s) = \frac{\Theta(s)}{V(s)} = \frac{K}{s(Ts+1)} = \frac{A}{s(s+B)} \quad (14)$$